Rolling motion of a ball spinning about a near vertical axis

A ball that is projected forward without spin on a horizontal surface will slide for a short distance before it starts rolling. Sliding friction acts to decrease the translation speed, v, and it acts to increase the rotation speed, ω. When $v = R\omega$, where R is the ball radius, the ball will start rolling and the friction force drops almost to zero since the contact point at the bottom of the ball comes to rest on the surface. The coefficient of rolling friction is much smaller than that for sliding friction. A different situation arises if the ball is projected forward while it is spinning about a vertical or near vertical axis. The latter situation arises in many ball sports. It arises if a player attempts to curve a ball down a bowling alley[1], or when a billiards player imparts sidespin or "English" to a ball[2] and it can arise in golf if a player strikes a ball with a putter at a point well away from the middle of the putter head. The situation also arises in the game of curling[3], although in that case the object that is projected is a cylindrical rock rather than a spherical ball, and it arises in tennis when a ball lands on the court spinning about a vertical axis (as it does in a slice serve).

A simple demonstration is to place a ball on a horizontal surface and spin it about a vertical axis using the thumb of one hand and the index finger of the other hand. The ball can be made to spin on the spot or it can be projected forward, depending on the motion of the two hands. When I tried this by projecting a golf ball forward on low pile carpet, I was surprised to find that (a) the spin axis remained almost vertical the whole time without the ball rolling forward as it came to a stop, (b) the ball kept spinning about a vertical axis after the forward motion stopped, the spin persisting for a longer time when the initial angular velocity was increased, and (c) the ball curved to the left or right depending on the direction of the initial spin. A literature search failed to provide explanations, so I filmed the ball at 300 frames/sec with a Casio EX-F1 video camera to obtain quantitative data. The data presented me with another surprise. The friction force on the ball was quite small, the same as that for a rolling ball. The ball therefore rolled to a stop, despite the fact that the ball kept spinning after the forward motion stopped and despite the fact that the ball appeared visually to slide on the carpet without rolling in the conventional manner.

Figure 1 shows the geometry of the experiment and a plausible explanation for the low friction force. Figure 1(a) is a bird's–eye view of a ball spinning about a vertical axis as it moves forward in the x direction. The ball curves to the right if it spins counter–clockwise viewed from above, the curvature increasing with the initial spin. In practice, it is difficult to spin a ball by hand so that the spin axis is exactly vertical. Figure 1(b) shows the spin axis inclined at an angle θ to the horizontal, as viewed front–on with the ball approaching the viewer. The contact point on the horizontal surface is directly below the center of the ball, and is located a perpendicular distance r from the spin axis. The contact point therefore rotates at speed $r\omega$ away from the viewer, relative to the center of mass, while the center of mass itself approaches the viewer at speed v. If $v = r\omega$ then the contact point is at rest on the surface. The ball can therefore roll forward along a circular path around the ball of radius r. For example, if $\theta = 70°$ then $r/R = \cos\theta = 0.34$ and if $\theta = 80°$ then $r/R = 0.17$.

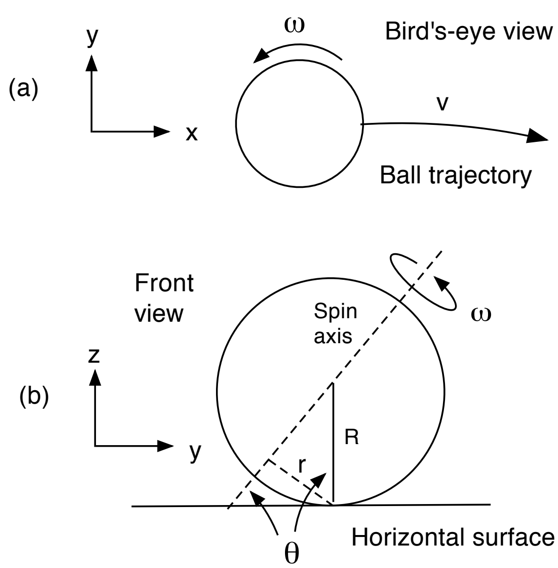

Figure 1: Geometry of a ball rolling on a horizontal surface at speed v in the x direction while the spin axis remains nearly vertical, viewed (a) from above and (b) from the front of the approaching ball. The z axis is vertical.

Figure 2 shows v and ω vs time for a typical case where the golf ball rolled forward over a distance of 72 cm before rolling to a stop. Also shown is the value of θ required for rolling for this ball (R = 22 mm). Both v and ω decreased linearly with time, but at slightly different rates. The linear deceleration of the ball was $a = 0.50 \pm 0.01$ m/s^2, giving a coefficient of friction $\mu = a/g = 0.051 \pm 0.001$. The angular deceleration, α, was 54 ± 1 rad/s^2. In separate experiments, it was found that $\mu = 0.044 \pm 0.002$ when the ball rolled to a stop without initial spin, $\mu = 0.33 \pm 0.01$ when the ball was sliding (struck with a cue stick to slide before it rolled) and $\alpha = 45 \pm 1$ rad/s^2 for a ball spinning on the spot. There was no significant change in deceleration rates when the ball was both spinning and translating, indicating that the ball did indeed roll when it was given initial spin.

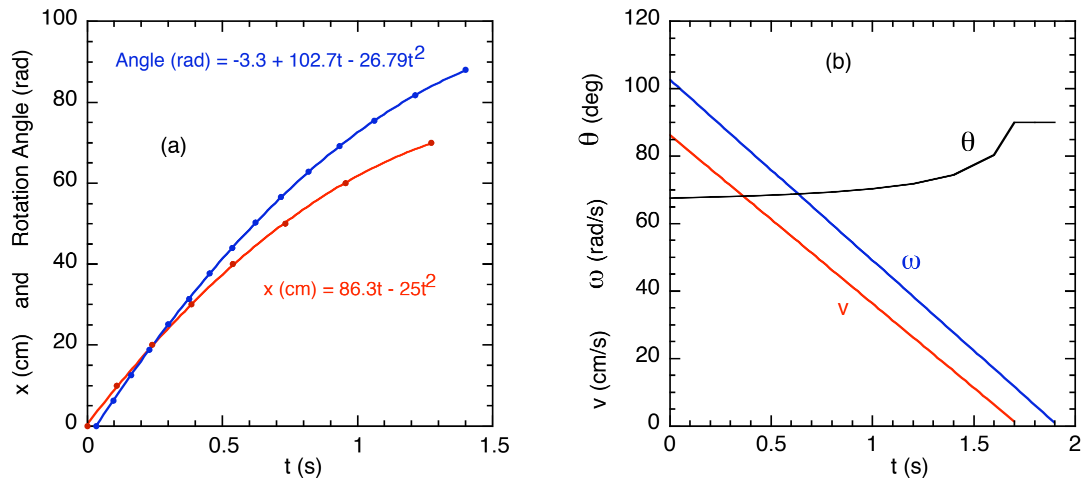

Figure 2: Experimental results for a golf ball projected at an initial speed $v = 86$ cm/s, and with initial spin $\omega = 103$ rad/s. (a) Raw data showing the x coordinate of the ball at 10 cm intervals and the rotation angle of the ball at intervals of one revolution. Quadratic fits were used to calculate v and ω vs time, as shown in (b). θ is the calculated inclination of the spin axis needed for the ball to roll. Note that the ball continued to spin for a short time after it stopped rolling.

The inclination of the spin axis could not be measured accurately, but the calculated value of θ shown in Fig. 2 is consistent with observations from the video film that the spin axis was about 20° away from the vertical at the start and much closer to the vertical when v decreased to zero. Two perpendicular lines were drawn around the ball circumference in order to measure ω accurately, but the lines did not accurately define the spin axis. A line near the equator for the spin axis wobbles slightly if it does not coincide exactly with the equator. A dot placed near the spin axis traces out a circular path around the spin axis, but it is difficult to locate a dot in this manner when spinning a ball by hand. The angular velocity of the ball is relatively easy to measure, despite the fact that the spin axis is difficult to identify unless it is almost vertical or almost horizontal. If the spin axis remains fixed, then the pattern of lines (and several dots) marked on the ball re–appears in the same orientation once every revolution, even though the pattern rotates simultaneously in vertical and horizontal directions in a manner that is difficult to interpret. If the spin axis rotates slowly then the pattern observed on the ball also rotates slowly every revolution.

It would seem too coincidental that the ball axis was inclined at exactly the right angle to commence rolling at the start. Presumably there was an initial sliding phase that quickly increased or decreased v (depending on whether the bottom of the ball was sliding backward or forward) so that the ball commenced rolling almost immediately. From then on, the spin axis slowly tilted into a vertical position to maintain the rolling condition.

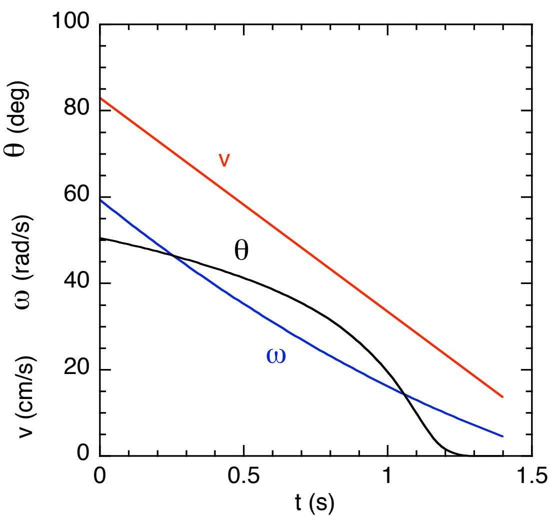

Figure 3: Experimental results for a golf ball projected at an initial speed $v = 83$ cm/s, and with initial spin $\omega = 59$ rad/s. Polynomial fits to the raw data were used to calculate v and ω vs time. θ is the calculated inclination of the spin axis needed for the ball to roll.

An additional experiment was performed where the ball was given a relatively small initial spin about a near vertical axis, a situation that is commonly encountered in billiards and golf and tenpin bowling. If a ball is projected with zero initial spin, then it quickly starts rolling about a horizontal axis. If the ball is projected with a small amount of spin about a vertical axis, then one might expect that the ball will stop spinning about its vertical axis before it stops translating. In that case, the ball should then commence to roll about a horizontal axis. That is essentially what happened, but the spin axis rotated with time into a horizontal position before the ball came to a stop. Results are shown in Fig. 3 for a golf ball projected on carpet with an initial forward speed $v = 83$ cm/s and an initial angular velocity $\omega = 59$ rad/s. In that case, v decreased at 0.49 m/s^2 giving $\mu = 0.050$, consistent with rolling friction. The spin decreased initially at the same rate as that found previously, with $\alpha = 53$ rad/s^2, although the angular deceleration decreased slightly as the ball slowed down. v and ω both decreased to zero at $t = 1.62$ s. The inclination of the spin axis decreased with time to maintain the rolling condition, according to the relation $\cos\theta = r/R = v/(\omega R)$, and it decreased to zero shortly before the ball came to rest. The inclination of the spin axis shown in Fig. 3 is consistent with visual observations of the ball from the slow motion video film.

The spin axis can therefore rotate either into a horizontal position or into a vertical position, depending on the ratio of v to ω at the start and on the rates at which v and ω separately decrease to zero. An analogous result is obtained with a spinning top, although the angle of inclination of a spinning top is not as easy to calculate since the top precesses and since the center of mass of a top is not directly above the point of support. Nevertheless, a spinning top rights itself when spinning rapidly and it falls to a horizontal position when spinning slowly[4]. The latter process is much more complicated, and has therefore received much more attention than the motion of a humble spherical ball. The question as to whether a disk or coin slides or rolls when it precesses is discussed in Ref. 5.

References

[1] C. Frohlich, "What makes bowling balls hook?," Am. J. Phys. **72**, 1170-1177 (Sept. 2004).

[2] S. Mathaven, M. Jackson and R. Parkin, "Application of high–speed imaging to determine the dynamics of billiards," Am. J. Phys. **77**, 788–794 (Sept. 2009).

[3] A.R. Penner, "The physics of sliding cylinders and curling rocks", Am. J. Phys. **69**, 332-339 (March 2001).

[4] D. Parkyn, "The rising of tops with rounded pegs, Physica", **24**, 313–330 (1958).

[5] D. Petrie, J. Hunt and C. Gray, "Does the Euler disk slip during its motion?", Am. J. Phys. **70**, 1025–1028 (Oct. 2002).